SUPPLEMENTARY MATERIAL

Detector Moments

All detector moments can be determined through conditioned characteristic functions,

$$ {}_f\langle e^{i\lambda x}\rangle = \frac{\mathrm{Tr}[(\hat{P}_f \otimes e^{i\lambda\hat{x}})\,\hat{U}_g\hat{\rho}_{SD}\hat{U}_g^\dagger]}{\mathrm{Tr}[(\hat{P}_f \otimes \hat{1}_D)\,\hat{U}_g\hat{\rho}_{SD}\hat{U}_g^\dagger]}, \quad (1a)$$

$$ = \frac{\mathrm{Tr}_S[\hat{P}_f\, e^{i\lambda g\hat{A}}\,\mathcal{X}_\lambda(\hat{\rho}_{SD})]}{\mathrm{Tr}_S[\hat{P}_f\,\hat{\rho}'_S]},$$

$$ {}_f\langle e^{i\lambda p}\rangle = \frac{\mathrm{Tr}[(\hat{P}_f \otimes e^{i\lambda\hat{p}})\,\hat{U}_g\hat{\rho}_{SD}\hat{U}_g^\dagger]}{\mathrm{Tr}[(\hat{P}_f \otimes \hat{1}_D)\,\hat{U}_g\hat{\rho}_{SD}\hat{U}_g^\dagger]}, \quad (1b)$$

$$ = \frac{\mathrm{Tr}_S[\hat{P}_f\,\mathcal{P}_\lambda(\hat{\rho}_{SD})]}{\mathrm{Tr}_S[\hat{P}_f\,\hat{\rho}'_S]},$$

where we have used the Weyl relation [1], $e^{ia\hat{x}}e^{-ib\hat{p}/\hbar} = e^{iab}e^{-ib\hat{p}/\hbar}e^{ia\hat{x}}$, and have defined the post-interaction reduced state $\hat{\rho}'_S = \mathrm{Tr}_D[\hat{U}_g\hat{\rho}_{SD}\hat{U}_g^\dagger]$, as well as the λ-dependent operations,

$$\mathcal{X}_\lambda(\hat{\rho}_{SD}) = \frac{1}{2}\mathrm{Tr}_D[\hat{U}_g(e^{i\lambda\hat{x}}\hat{\rho}_{SD} + \hat{\rho}_{SD}e^{i\lambda\hat{x}})\hat{U}_g^\dagger], \quad (2a)$$

$$\mathcal{P}_\lambda(\hat{\rho}_{SD}) = \frac{1}{2}\mathrm{Tr}_D[\hat{U}_g(e^{i\lambda\hat{p}}\hat{\rho}_{SD} + \hat{\rho}_{SD}e^{i\lambda\hat{p}})\hat{U}_g^\dagger]. \quad (2b)$$

Computing derivatives of the characteristic functions produces the conditioned detector moments,

$$ {}_f\langle x^n\rangle = \frac{\partial^n}{\partial(i\lambda)^n}\,{}_f\langle e^{i\lambda x}\rangle\Big|_{\lambda=0}, \quad (3a)$$

$$ {}_f\langle p^n\rangle = \frac{\partial^n}{\partial(i\lambda)^n}\,{}_f\langle e^{i\lambda p}\rangle\Big|_{\lambda=0}. \quad (3b)$$

This procedure is similar in spirit to the full counting statistics approach employed in [2].

The first two moments are given explicitly by,

$$ {}_f\langle x\rangle = \mathrm{Re}\,\langle x\rangle^w + g\,\mathrm{Re}\,\langle A\rangle^w, \quad (4a)$$

$$ {}_f\langle p\rangle = \mathrm{Re}\,\langle p\rangle^w, \quad (4b)$$

$$ {}_f\langle x^2\rangle = \mathrm{Re}\,\langle x^2\rangle^w + 2g\,\mathrm{Re}\,\langle xA\rangle^w + g^2\,\mathrm{Re}\,\langle A^2\rangle^w, \quad (4c)$$

$$ {}_f\langle p^2\rangle = \mathrm{Re}\,\langle p^2\rangle^w, \quad (4d)$$

in terms of the Heisenberg evolved joint post-selection $\hat{P}'_{SD} = \hat{U}_g^\dagger(\hat{P}_f \otimes \hat{1}_D)\hat{U}_g$ and the joint weak values,

$$\langle x\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{1}_S\otimes\hat{x})\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5a)$$

$$\langle A\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{A}\otimes\hat{1}_D)\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5b)$$

$$\langle p\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{1}_S\otimes\hat{p})\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5c)$$

$$\langle x^2\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{1}_S\otimes\hat{x}^2)\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5d)$$

$$\langle Ax\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{A}\otimes\hat{x})\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5e)$$

$$\langle A^2\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{A}^2\otimes\hat{1})\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}, \quad (5f)$$

$$\langle p^2\rangle^w = \frac{\mathrm{Tr}[\hat{P}'_{SD}\,(\hat{1}_S\otimes\hat{p}^2)\,\hat{\rho}_{SD}]}{\mathrm{Tr}[\hat{P}'_{SD}\,\hat{\rho}_{SD}]}. \quad (5g)$$

Detector Wigner Function

Assuming an initial product state $\hat{\rho}_{SD} = \hat{\rho}_S \otimes \hat{\rho}_D$, we can compute the operations (2) as follows. After computing the detector trace in the p-basis and inserting two complete x-basis sets, the $\mathcal{P}_\lambda$ operation takes the form

$$\begin{aligned}\mathcal{P}_\lambda(\hat{\rho}_{SD}) &= \iiint \frac{dp\,dx\,dx'}{2\pi\hbar}\,\langle x'|\hat{\rho}_D|x\rangle \\ &\qquad e^{-i\frac{p}{\hbar}(x-x'-g\,\mathrm{ad}[\hat{A}]+\hbar\lambda)}(\hat{\rho}_S), \qquad (6)\\ &= \iint dx\,dx'\,\langle x'|\hat{\rho}_D|x\rangle \\ &\qquad \delta(x-x'-g\,\mathrm{ad}[\hat{A}]+\hbar\lambda)(\hat{\rho}_S), \\ &= \int dz\,\widetilde{W}_D(z, g\,\mathrm{ad}[\hat{A}]-\hbar\lambda)(\hat{\rho}_S).\end{aligned}$$

Here we have changed integration variables to $z = x - x'$ and $y = (x + x')/2$, and have noted that $\widetilde{W}_D(z, y) = \langle z - y/2|\hat{\rho}_D|z + y/2\rangle$ is the Fourier-transformed Wigner function of the detector.

Performing a similar computation for $\mathcal{X}_\lambda$ yields,

$$\begin{aligned}\mathcal{X}_\lambda(\hat{\rho}_{SD}) &= \iiint \frac{dp\,dx\,dx'}{2\pi\hbar}\,\frac{1}{2}(e^{i\lambda x'} + e^{i\lambda x})\langle x'|\hat{\rho}_D|x\rangle \\ &\qquad e^{-i\frac{p}{\hbar}(x-x'-g\,\mathrm{ad}[\hat{A}])}(\hat{\rho}_S), \qquad (7)\\ &= \iint dx\,dx'\,\frac{1}{2}(e^{i\lambda x'} + e^{i\lambda x})\langle x'|\hat{\rho}_D|x\rangle \\ &\qquad \delta(x-x'-g\,\mathrm{ad}[\hat{A}])(\hat{\rho}_S), \\ &= \int dz\,e^{i\lambda z}\widetilde{W}_D(z, g\,\mathrm{ad}[\hat{A}])\cos\left(\frac{\lambda g}{2}\mathrm{ad}[\hat{A}]\right)(\hat{\rho}_S).\end{aligned}$$

Taking derivatives with respect to $(i\lambda)$ produces the expressions in the main text for the first moments. Setting $\lambda = 0$ in either $\mathcal{P}_\lambda(\hat{\rho}_S)$ or $\mathcal{X}_\lambda(\hat{\rho}_S)$ produces the post-interaction reduced system state $\hat{\rho}'_S$.

The operation $\mathrm{ad}[\hat{A}]$ is linear, so any analytic function of $\mathrm{ad}[\hat{A}]$ may be defined via its Taylor series in the same manner as an analytic function of a matrix. Indeed, to more rigorously perform the above derivations one can exploit an isomorphism that maps $\hat{\rho}_S$ into a vector and $\mathrm{ad}[\hat{A}]$ into a matrix acting on that vector. After expanding the expressions into the eigenbasis of the matrix of $\mathrm{ad}[\hat{A}]$ and regularizing any singular functions into limits of well-behaved analytic functions, the above computations can be performed for each eigenvalue, summed back into a matrix, and then mapped back into the operator form shown. For unbounded $\hat{A}$ then one must also carefully track the domains to ensure that the resulting expressions properly converge, as they should for any physically sensible result.

Hermite-Gauss superpositions

The Wigner distribution for an arbitrary superposition of Hermite-Gauss modes $|\psi\rangle = \sum_m c_m |h_m\rangle$ can be computed to find (suppressing arguments for compactness),

$$W = \sum_{m,n=0}^{\infty} \frac{c_m c_n^*}{\sqrt{m!\,n!}} \frac{(-1)^m\, e^{i(m-n)\phi}}{\pi\hbar} D_n^m[\sqrt{2G}]\, e^{-G}, \quad (8)$$

where $G(x,p) = x^2/2\sigma^2 + 2p^2\sigma^2/\hbar^2$ and $\phi(x,p) = \tan^{-1}(-2p\sigma^2/\hbar x)$. The polynomial sequence $D_n^m(x)$ has the generating function,

$$\exp(z\bar{z} - x(z - \bar{z})) = \sum_{m,n=0}^{\infty} \frac{z^m \bar{z}^n}{m!\,n!} D_n^m(x), \quad (9)$$

and the explicit form,

$$D_n^m(x) = \sum_{k=0}^{\min(m,n)} \frac{m!\,n!\,(-1)^{m-k}}{(m-k)!\,(n-k)!\,k!}\, x^{m+n-2k}. \quad (10)$$

Notably, the diagonal elements of this sequence are the Laguerre polynomials, $D_m^m(x) = m!\,L_m(x^2)$. These results can be obtained by using the generating function for the Hermite polynomials $\exp(2xz - z^2) = \sum_{n=0}^{\infty} H_n(x) z^n/n!$, as well as the identities $H_n(x) = \exp(-\partial_{2x}^2)(2x)^n$ and $\int dx\, e^{-x^2} H_n(x) H_m(x) = \delta_{m,n}\, m!\,\sqrt{\pi}\, 2^m$.

Computing the reduced system state $\hat{\rho}'_S$ using this Wigner function yields,

$$\hat{\rho}'_S = \sum_{m,n=0}^{\infty} \frac{c_m c_n^*}{\sqrt{m!\,n!}} D_n^m\left[\sqrt{\epsilon}\,\mathrm{ad}[\hat{A}]\right] e^{\epsilon\mathcal{L}[\hat{A}]}(\hat{\rho}_S), \quad (11)$$

where $\epsilon = (g/2\sigma)^2$, and $\mathcal{L}[\hat{A}] = -\mathrm{ad}^2[\hat{A}]/2$ is the Lindblad decoherence operation. Notably, the functional form of the Wigner distribution (8) is still largely preserved in Eq. (11).

The detector averages can also be computed from this Wigner function. The weak value $\mathrm{Re}\,\langle x\rangle^w$ will vanish by symmetry; the weak value $\mathrm{Re}\,\langle p\rangle^w$ involves the derivative $i\hbar\partial_{g\mathrm{ad}[\hat{A}]}\hat{\rho}'_S$; and, the weak value $\mathrm{Re}\,\langle A\rangle^w$ involves the state $\hat{\rho}'_S$ directly. When $c_m = 1$ with the rest of the coefficients zero, then these generalizations reduce to the results presented in the main text.

[1] R. Alicki and M. Fannes, *Quantum dynamical systems* (Oxford University Press, 2001).
[2] A. Di Lorenzo, Phys. Rev. A **85**, 032106 (2012).